ABSTRACT
of the dissertation submitted by Yuliya Sabirova for the degree of Doctor of Philosophy (PhD) in the educational program
«8D06104 – Mathematical and Computer Modeling» entitled «Modelling of dynamics and vibration protection of nonlinear deformable systems»

General Characteristics of the Study. This work is devoted to the modeling of the nonlinear dynamics of deformable systems and the development of methods for their analysis, with application to problems associated with drilling equipment in the oil and gas industry. The influence of material parameter nonlinearities and structural layering on the performance of vibration protection systems is investigated.

Relevance of the Research Topic. Contemporary research is focused on addressing nonlinear problems in deformable systems. The nonlinearity of such systems arises from the removal of restrictions on the magnitude of deformations, the physical nonlinearity of structural materials, nonlinear inertia in dynamic systems, and other factors. This necessitates the development of new mathematical models and methods for their analysis, as well as computational algorithms and software. All these advances will enable applied problems to be addressed at a new qualitative level.

These studies are particularly relevant in the context of developing vibration protection systems for drilling equipment used in the oil and gas industry to prevent undesirable vibrations and premature wear of components. Of particular interest is the influence of the physically nonlinear properties of structural elements, such as dampers, on the nonlinear dynamics of drill strings. Advances in modern materials science are driving the transition toward the use of complex multilayer composite structures as highly efficient dampers. The widespread application of such multifunctional, high-contrast composites in civil, industrial, aerospace, and automotive engineering requires refinement of the existing fundamental theory, which was developed for homogeneous and/or weakly heterogeneous structures, since practically important boundary effects observed in high-contrast laminates require further investigation.

Research Aim. Development of mathematical modeling methods for the nonlinear dynamics of deformable systems, including rod elements of drill strings and thin-walled layered damping sleeves, aimed at improving the efficiency of vibration protection systems.

In accordance with this aim, the following **research objectives** are formulated:

– to develop generalized nonlinear dynamic models of rod systems, taking into account vibration protection devices;

– to develop a software and computational package for the numerical implementation of the proposed models, ensuring high accuracy in the analysis of the dynamic characteristics of heterogeneous structures;

– to verify the developed software and computational package by comparison with results available in the literature;

– to conduct a multiparametric analysis of the nonlinear dynamics of drill strings, taking into account the influence of external factors, and to identify the operating patterns of vibration protection devices under various operating conditions;

– to perform an asymptotic analysis of boundary antiplane problems for high-contrast layered plates and plates on a compliant nonlinear Winkler-type foundation.

Research Object. The object of the dissertation research is nonlinear deformable systems, in particular, rod elements of drilling equipment and elastic layered plates.

The subject of the research is the mathematical modeling of problems in the mechanics of deformable bodies arising in the development of vibration protection methods for drilling equipment.

Research Methods. The study employs an integrated approach based on fundamental principles of mechanics, including the Ostrogradsky-Hamilton variational principle for deriving

the equations of motion and the nonlinear theory of deformation developed by V. V. Novozhilov; modern methods of mathematical modeling of media and materials, including the use of the Treloar elastic potential to describe physically nonlinear elements and the theory of thin elastic plates and rods; analytical methods, including asymptotic analysis and the Fourier method for solving elliptic equations and determining decay conditions; numerical and discrete methods, including the lumped-parameter method and the Runge-Kutta method; and computational modeling techniques implemented using C++, Python, and OpenMP.

Scientific Novelty of the Study. The scientific novelty of the work lies in the development of refined nonlinear dynamic models of rod systems that, for the first time, account for the coupling between rod displacements and the intrinsic kinematics of damping elements modeled as geometrically and physically nonlinear continuous deformable bodies. An original algorithmic and software framework has been developed in C++ using parallel computing, adapted for the analysis of the spatial motion of drill strings under complex loading conditions, including torsion, axial compression, and fluid flows. For the first time, based on an asymptotic method, refined boundary conditions (the Saint-Venant principle) have been derived for antiplane shear problems in high-contrast layered plates and plates on a nonlinear elastic Winkler foundation.

Scientific Contributions Submitted for Defense

– a set of nonlinear mathematical models of rod systems accounting for variable structural configurations and coupling with continuous damping elements. It is substantiated that accounting for the intrinsic kinematics of vibration protection devices makes it possible to identify dynamic effects that cannot be described within the framework of conventional discrete models;

– an algorithmic implementation and software package based on the lumped-parameter method using parallel programming technologies. The developed architecture enables the efficient solution of large-scale problems for systems subjected to complex loading conditions, including rotation, fluid flows, and nonlinear sleeves, while ensuring high computational stability;

– results of a multiparametric analysis of drill string dynamics, establishing relationships between the geometric parameters of stabilizers, the characteristics of nonlinear sleeve, and the dynamic characteristics of the system. The effectiveness of rubber-cord elements for damping lateral vibrations under conditions of supersonic flow and axial compression has been demonstrated;

– a generalization of the Saint-Venant principle for antiplane shear problems applied to three-layer high-contrast plates and homogeneous plates supported by a compliant nonlinear Winkler-type foundation.

Reliability and Validity of the Scientific Provisions, Conclusions, and Results of the Dissertation are ensured by the use of well-established fundamental laws of continuum mechanics. The mathematical models were developed in accordance with established methodologies widely applied in practice. The developed software package was verified using benchmark problems and by comparison with results available in the literature. The parametric analysis and energy plots demonstrate physically consistent and well-grounded results.

Theoretical and Practical Significance of the Research. The development of new and the generalization of existing mathematical models of nonlinear deformable systems will expand the current understanding of the nature of vibrations and contribute to the theoretical knowledge of solid deformation. The development of a unified methodology for analyzing such systems will improve the accuracy of engineering calculations and broaden the range of problems that can be addressed in the mechanics of deformable media. The results of this study have broad practical applications in the analysis of the dynamics of machines and mechanisms.

Relationship of the Research to Other Scientific Projects. The dissertation research was carried out within the framework of three scientific projects funded under the Grant Financing Program of the Ministry of Education and Science of the Republic of Kazakhstan (Projects 1–3), as well as within the framework of a joint British-German research project involving Keele University and German Aerospace Center (DLR) (Project 4).

1) «Development of Mathematical Models and Analysis Methods for Mechanical Deformable Systems with Complex Nonlinear Properties and Variable Structure in Machine Dynamics» **(2020-2022 гг., AP08857255-OT-22);**

2) «Development of Nonlinear Mathematical Models and Software Modules for Solving Drilling Problems in the Extractive Industry» **(2021-2023 гг., AP09261135-OT-23).**

3) «Modeling the Dynamics of Nonlinearly Deformable Systems and Heterogeneous Nanostructures under Complex Conditions for Solving Problems in the Extractive Industry» **(2024-2026 гг., AP23490543)**

4) Asymptotic analysis of boundary value problems for strongly inhomogeneous multi-layered elastic plates **(2024-2026, EP/Y021983/1)**

Research Dissemination. The research results were presented and discussed at the following scientific events:

– 58th International Scientific Student Conference "ISSC-2020 (MNSC-2020)", Novosibirsk, Russia, 2020;

– XLVIII International Summer School-Conference "Advanced Problems in Mechanics", Saint Petersburg, Russia, 2020;

– 16th International Conference "Dynamical Systems – Theory and Applications", Łódź, Poland, 2021;

– III International Student Scientific and Practical Conference "Youth Digital Forum (YDF-2022)", Almaty, Kazakhstan, 2022;

– VIII International Scientific and Practical Conference "Informatics and Applied Mathematics", dedicated to the memory of Professor R. G. Biyashev, Doctor of Technical Sciences, Almaty, Kazakhstan, 2023;

– International Conference "Computational and Information Technologies in Science, Engineering and Education", dedicated to the 90th anniversary of Al-Farabi Kazakh National University and the 75th anniversary of Academician Sh. S. Smagulov, Almaty, Kazakhstan, 2024;

– The 5th International Conference on Nonlinear Dynamics NODYCON 2026 (Rome, Italy, 2026);

– Research seminars of the Mechanics and Mathematics Faculty of Al-Farabi Kazakh National University (2020–2026, Almaty, Kazakhstan);

– Research seminars of the Department of Mathematical and Computer Modeling, Al-Farabi Kazakh National University (2020–2026, Almaty, Kazakhstan).

Publications. A total of 11 publications have been published on the topic of the dissertation, including: 3 papers published in journals indexed in the Scopus and Web of Science databases: Zeitschrift für angewandte Mathematik und Physik (SJR=0.806, CiteScore percentile=81, Q1) [1], Applied Mathematical Modelling (AMM) (SJR = 1.126, CiteScore percentile = 97, Q1) [3] and Symmetry (SJR = 0.467, CiteScore percentile = 93, Q2) [4]; 1 paper published in a journal indexed in the Scopus and Web of Science databases and included in the list of publications recommended by the Committee for Quality Assurance in Science and Higher Education (CQASHE) of the Ministry of Science and Higher Education of the Republic of Kazakhstan [5]; 7 conference publications in the proceedings of international conferences [2, 6-11], including 4 publications presented at international conferences held abroad [2, 9-11].

1. Kaplunov J., Prikazchikova L., **Sabirova Yu.,** Erbaş B. A refined decay condition for a high-contrast sandwich plate subject to antiplane shear // Zeitschrift für angewandte Mathematik und Physik. – 2026. – Vol. 77. – No 10. – Art. 281.
2. **Sabirova Yu.**, Khajiyeva L., Sabirova R. Impact of a Continuous Damping Sleeve on Nonlinear Drill String Dynamics in Fluid Flow // NODYCON 2026: Fifth International Nonlinear Dynamics Conference: Book of Abstracts. – Rome, Italy : Sapienza University of Rome, 2026. – P. 156.
3. Khajiyeva L., **Sabirova Yu.,** Kudaibergenov Askar, Kudaibergenov Askat. Application of the lumped-parameter method for modeling nonlinear vibrations of drill strings with

stabilizers in a supersonic gas flow // Applied Mathematical Modelling. – 2022. – Vol. 110. – P. 748–766.

4. Khajiyeva L., Andrianov I., **Sabirova Yu.,** Kudaibergenov A.. Analysis of Drill String Nonlinear Dynamics Using the Lumped-Parameter Method // Symmetry. – 2022. – Vol.14. – No 7. – Art. 1495.
5. Khajiyeva L., **Sabirova Yu.,** Sabirova R. Modelling of horizontal drill string motion by the lumped-parameter method // Вестник КазНУ. Серия математика, механика, информатика. – 2022. – Т. 115, № 3. С. 132-142.
6. **Sabirova Yu.,** Prikazchikova L. Decay conditions for antiplane shear of a high-contrast sandwich plate // Proc. Int. Conf. “Computational and Information Technologies in Science, Engineering and Education”. – Almaty, 2024. – P. 215.
7. **Sabirova Yu.,** Sabirova R. Development of a numerical model of a drill string in a fluid flow by the lumped-parameter method // Proc. VIII Int. Sci.-Pract. Conf. “Informatics and Applied Mathematics”. – Almaty, 2024. – P. 101-106.
8. **Sabirova Yu.** Mathematical modeling and computer visualization of drill string dynamics using OpenGL // Proc. III Int. Student Sci.-Pract. Conf. “Youth Digital Forum YDF-2022 (YDF-2022)”. – Almaty, 2022. – P. 263-264 (in Russian).
9. Khajiyeva L., Kudaibergenov A., **Sabirova Yu..** Application of the lumped parameter method for modelling nonlinear vibrations of drill strings with complicating factors // Abstracts 16th Int. Conf. “Dynamical Systems – Theory and Applications (DSTA 2021)”. – Lodz, Poland, 2021. – P. 751-752 (online).
10. Khajiyeva L., Kudaibergenov A., **Sabirova Yu..** Modeling of dynamics of a drill string with variable structure by the lumped parameters method // Abstracts XLVIII Int. Summer School-Conf. “Advanced Problems in Mechanics”. – St. Petersburg, Russia, 2020. – P. 57-58 (online).
11. **Sabirova Yu.** Numerical modeling of dynamic systems with variable structure by the lumped-parameter method // Proc. 58th Int. Sci. Conf. – Novosibirsk, Russia, 2020. – P. 99. (in Russian).

Author’s Contribution. All results presented in the dissertation were obtained independently by the author under the supervision of the academic advisors. The author’s personal contribution includes the development and verification of a new software package, the development of generalized models presented in Section III, the asymptotic analysis of the problems presented in Section IV, the dynamic and multiparametric analyses, as well as the generation and analysis of plots for the problems considered in all sections of the dissertation.

Structure and Volume of the Dissertation. The dissertation consists of a title page, table of contents, normative references, a list of symbols and abbreviations, an introduction, four sections, a conclusion, a list of references comprising 128 sources, and 3 appendices. The dissertation comprises 183 pages and contains 90 figures and 18 tables.

Main Content of the Dissertation. The Introduction presents a general overview of the dissertation, including the aim and objectives of the study, the research methods employed, the theoretical and practical significance of the work, publications, projects within which the research was conducted, and a list of events at which the results of the work were presented and discussed.

Chapter I provides an analytical review of the current state of research in the field of nonlinear dynamics of deformable systems. The need to improve mathematical models of drill strings by accounting for the influence of vibration protection elements is substantiated. The relevance of studying low-frequency vibrations in high-contrast composites is also demonstrated, due to the insufficient understanding of wave processes in structures with substantially different layer parameters. The problem statements are formulated.

Chapter II is devoted to the development of a computational software package in C++ for modeling the planar and spatial nonlinear motion of drill strings as lumped-parameter systems. The software package is verified using established benchmark models.

Chapter III focuses on the study of the nonlinear dynamics of drill strings under complex dynamic loading, taking into account vibration protection elements, including support stabilizers and damping sleeves. The complex loading conditions include axial compression, torsion, self-weight, and the action of gas flow on a rotating rod. A distinctive feature of the study is that the intrinsic kinematics of the sleeve is taken into account by modeling it as a geometrically and physically nonlinear continuous body.

Chapter IV is devoted to the investigation of local dynamic processes in the contact zones between vibration protection elements and the main structural components. An antiplane shear problem for high-contrast layered plates is solved using asymptotic analysis within a refined boundary theory. To account for the interaction of damping elements with a nonlinear elastic medium, such as rubber-cord layers and the wellbore foundation, the problem is generalized to the case of a semi-infinite plate on a nonlinear Winkler foundation.

The Conclusion summarizes the main results of the study, assesses the completeness with which the stated objectives have been achieved, and provides recommendations for the practical application of the obtained results.