APPROVED
at a meeting of the Academic Council of NJSC «KazNU named after al-Farabi»
Protocol № 11 from 23.05.2025 y.

The program of the entrance exam for applicants to the PhD for the group of educational programs D092 – «Mathematics and statistics»

I. General provisions

1. The program was drawn up in accordance with the Order of the Minister of Education and Science of the Republic of Kazakhstan dated October 31, 2018 No. 600 «On Approval of the Model Rules for Admission to Education in Educational Organizations Implementing Educational Programs of Higher and Postgraduate Education» (hereinafter referred to as the Model Rules).

2. The entrance exam for doctoral studies consists of writing an essay, an exam in the profile of a group of educational programs and an interview.

Блок	Баллы
1. Interview	30
2. Essay	20
3. Exam according to the profile of the group of the educational program	50
Total admission score	100/75

3.The duration of the entrance exam is 3 hours 10 minutes, during which the applicant writes an essay and answers the electronic examination ticket. The interview is conducted at the university premises before the entrance exam.

II. Procedure for the entrance examination

1. Applicants for doctoral studies in the group of educational programs D092 – «Mathematics and statistics» write a problematic / thematic essay. The volume of the essay is at least 250words.

The purpose of the essay is to determine the level of analytical and creative abilities, expressed in the ability to build one’s own argumentation based on theoretical knowledge, social and personal experience.

Types of essays:

- motivational essay revealing the motivation for research activities;
- scientific-analytical essay justifying the relevance and methodology of the planned research;
- problem/thematic essay reflecting various aspects of scientific knowledge in the subject area.

2. The electronic examination card consists of 3 questions

Topics for exam preparation according to the profile of the group of the educational program:

Discipline "Mathematical Analysis"

Numerical sequences. Upper and lower limits. Bolzano-Weierstrass theorem and Cauchy criterion for numerical sequences. Limit of functions, continuity and uniform continuity of functions.

Weierstrass theorem on uniform continuity on a closed segment. Derivative and differential of a function of one variable. Relationship between them. Invariance of the form of the first differential. The concept of the inverse function and formulation of the question. Prove the simplest version of the theorem on the existence of the inverse function. Differentiation of the inverse function of one variable, derivatives of inverse trigonometric functions. Function of several variables. Multiple and iterated limits. Relationship between them. Partial derivatives. Differentiation of a function of several variables. Differentiability of functions of several variables. Differentiation of a composite function of several variables. The concept of an implicit function and formulation of the question. General theorem on implicit and inverse functions. Jacobian. Change of variables in a multiple integral. Green's formula for a double integral. Surface integrals. Fundamental theorems of integral calculus.

Course "Functional Analysis"

Metric, linear normed, Banach and Hilbert spaces. Examples of metric, normed, Banach and Hilbert spaces. Sequences and properties of convergent sequences in metric and linear normed spaces. Continuous mappings in metric space. Continuity and compactness in metric spaces. The principle of contracting mappings in metric space. General form of a linear bounded functional in a Hilbert space. Riesz's theorem. Measurable sets and their properties. Measurable functions and their properties. Lebesgue integral. Difference between Lebesgue and Riemann integrals. $Lp(\Omega)$ spaces and their properties. Linear operators in Banach and Hilbert spaces. Bounded operators, unbounded operators, closed operators. Norm of an operator.

Discipline "Probability Theory and Stochastic Analysis"

General probability space. Classical and geometric definition of probabilities. Conditional probability. Formula for the product of probabilities. Independent events, independent trials. Formula for total probability. Bayes' formula. Random variables. Laws of distribution of random variables. Mathematical expectation of random variables. Variance. Repeated independent trials. Bernoulli's formulas. General definition of a random process and finite-dimensional distributions of a random process. Wiener process. Finite-dimensional distributions of a Wiener process and characterization property of a Wiener process. Correlation function of a random process. Properties.

Course «Algebra and Geometry»

Concepts of algebraic structure. Homomorphisms and isomerisms of algebraic structures. Automorphism group of algebraic structures. Examples. Semigroups. Monoids. Invertible elements. Groups. Cyclic groups. Isomorphisms. Cayley's theorem. Homomorphisms. Kernel and image of a homomorphism. Relations with normal subgroups. Cosets. Indices. Lagrange's theorem and its consequences. Ring. Zero divisors. Congruences. Residue class ring. Homomorphisms of rings. Field. Characteristic of a field. Finite fields. Construction of a Galois field. Relations. Equivalence relations, properties of equivalence classes. Partial order relation. Linear order. Least, greatest, minimal and maximal elements. Prove that a finite partially ordered set always has a minimal element. Dirichlet's principle. Inclusion and exclusion formula. The number of elements in the Cartesian product of a finite number of finite sets.

Course "Differential Equations and Equations of Mathematical Physics"

Theorems of existence and uniqueness of the solution of the Cauchy problem for ordinary differential systems of first order. Homogeneous linear ordinary differential equation of the n-th order with variable coefficients. Fundamental system of solutions. Inhomogeneous linear ordinary differential equation of the n-th order with constant coefficients. Systems of homogeneous linear ordinary differential equations, properties of solutions. Ostrogradsky-Liouville formula. Statement of boundary value problems for a linear ordinary differential equation of the second order. Sturm-Liouville problem. Theorems of existence and uniqueness of the Sturm-Liouville solution. Existence of eigenvalues of boundary value problems for a linear ordinary differential equation. Definition of the Green's function for the Sturm-Liouville problem and its existence. Solution of boundary value problems for an ordinary differential equation using the Green's function. Inhomogeneous systems of linear differential equations. Method of variation of arbitrary constants (Lagrange method). Classification and reduction to canonical form of second-order partial differential equations in the case of many variables. Cauchy problem for a parabolic equation. Fundamental solution of the heat conductivity operator. Volume thermal potential, surface thermal potential and their main properties. Cauchy problem for a hyperbolic equation. Concept of a characteristic for a hyperbolic equation. Continuation method. Statement and main methods for solving boundary value problems for an equation of elliptic type. Hadamard's example on the ill-posedness of the Cauchy problem for the Laplace equation. Method of separation of variables. General scheme of the Fourier method. Eigenvalue and eigenfunction problem for the Sturm-Liouville operator. Fourier method for solving mixed problems for parabolic and hyperbolic equations. Cylindrical functions. Bessel equation. Bessel functions. Dirichlet and Neumann problems for the Laplace and Poisson equations. Green's function for the Dirichlet problem, its properties. Solution of the boundary value problem for the Poisson equation using the Green's function. Variation and its properties. Euler's equation. The fundamental lemma of the calculus of variations. The brachistochrone problem. The simplest problem of the calculus of variations with moving boundaries. The transversality condition. Sufficient conditions for a functional to attain an extremum. The Legendre condition. Variational problems for a conditional extremum. The concept of constraints. Reduction to a problem for an unconditional extremum. Lagrange multipliers. Weierstrass's theorem in a Banach space

III List of references

Main:

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