

## Brief information about the project

Name of the project	№ AP09058457 Development of research methods and solution of boundary value problems for loaded hyperbolic equations and their numerical implementation
Relevance	The project is aimed at studying boundary value problems for loaded hyperbolic equations, where the load point is set by a spatial variable. Loaded hyperbolic equations are used in problems of biology, chemistry, agricultural systems, etc. The problems for loaded hyperbolic equations have become particularly relevant in connection with the study of the vibration stability of aircraft wings loaded with masses and in calculating the natural vibrations of antennas loaded with concentrated capacitances and self-inductions. Despite a large number of works devoted to the study and solution of boundary value problems for loaded differential, integro-differential and hyperbolic equations, many issues related to the solvability of boundary value problems for linear loaded hyperbolic equations, for nonlinear loaded hyperbolic equations with mixed derivatives, where load points are set by a spatial variable remain unresolved, there are no methods that give to construct a numerical and analytical solution.
Purpose	The objectives of the project are:  - to develop constructive methods for researching and solving boundary value problems for linear and nonlinear loaded hyperbolic equations with mixed derivatives, where load points are set according to a spatial variable and to establish conditions for their solvability;  - to develop numerical and analytical methods for solving boundary value problems for loaded hyperbolic equations and systems with point loads in a spatial variable.
Objectives	1. To investigate the boundary value problem for a linear loaded hyperbolic equation: 1.1 Obtaining necessary and sufficient conditions for the correct solvability of the boundary value problem for linear loaded hyperbolic equations, where the load points are set according to a spatial variable. 1.2 Development of a numerical and approximate method for solving linear boundary value problems for loaded hyperbolic equations, where load points are set according to a spatial variable, based on the Euler polyline modification method. Establishing the properties of new general solutions; Obtaining a solvability condition. 1.3 Implementation of the constructed numerical algorithms for boundary value problems of linear loaded hyperbolic equations in the C++ and Python programming languages. 2. To investigate the boundary value problem for linear loaded systems of a hyperbolic equation:

	<p>2.1 Construction of a new general solution for a linear ordinary loaded differential equation and establishment of its properties.</p> <p>2.2 Development of a numerical and approximate method for solving linear boundary value problems for systems of loaded hyperbolic equations, where load points are set according to a spatial variable, based on the Euler polyline modification method and the establishment of conditions for its convergence.</p> <p>2.3 Implementation of the constructed numerical algorithms in the C++ and Python programming languages.</p> <p>3. To investigate the boundary value problem for a nonlinear loaded hyperbolic equation:</p> <p>3.1 Obtaining sufficient conditions for the existence of an isolated solution of the boundary value problem for a nonlinear loaded hyperbolic equation, where the load points are set according to a spatial variable.</p> <p>3.2 Solving a family of boundary value problems for nonlinear loaded hyperbolic equations based on the parametrization method and obtaining their solvability condition.</p> <p>3.3 Development of numerical and approximate methods for solving non-local boundary value problems for nonlinear loaded hyperbolic equations with mixed derivatives, where load points are set according to a spatial variable, based on modifications of the Euler polyline method and methods for solving nonlinear boundary value problems for nonlinear ordinary loaded differential equations and to establish conditions for their solvability.</p>
<p>Expected and achieved results</p>	<p>In this project, boundary value problems for linear, nonlinear loaded hyperbolic equations and for linear systems of loaded hyperbolic equations and their numerical implementation will be studied. The expected results coincide with the tasks of the project.</p> <p>The following results are obtained:</p> <ul style="list-style-type: none"> <li>- necessary and sufficient conditions for the correct solvability of the boundary value problem for linear loaded hyperbolic equations are obtained, where the load points are set by a spatial variable;</li> <li>- a numerical and approximate method for solving linear boundary value problems for loaded hyperbolic equations based on the Euler polyline modification method has been developed, properties of new general solutions have been established and solvability conditions have been obtained;</li> <li>- algorithms for solving boundary value problems of linear loaded hyperbolic equations are implemented;</li> <li>- general solutions for a linear ordinary loaded differential equation are constructed and its properties are established;</li> <li>- numerical methods for solving linear boundary value problems for systems of loaded hyperbolic equations have been developed, the conditions for its convergence have</li> </ul>

	<p>been established and the constructed numerical algorithms have been implemented;</p> <ul style="list-style-type: none"> <li>- sufficient conditions for the existence of an isolated solution of the boundary value problem for a nonlinear loaded hyperbolic equation are obtained;</li> <li>- a method for solving a family of boundary value problems for nonlinear loaded hyperbolic equations has been developed and conditions for their solvability will be obtained;</li> <li>- a numerically approximate method for solving boundary value problems of nonlinear loaded hyperbolic equations with mixed derivatives has been developed;</li> <li>- a method for solving nonlinear boundary value problems for nonlinear ordinary loaded differential equations has been developed and the conditions for their solvability have been established.</li> </ul>
<p>Research team members with their identifiers (Scopus Author ID, Researcher ID, ORCID, if available) and links to relevant profiles</p>	<ol style="list-style-type: none"> <li>1. Kabdrakhova Symbat Seisenbekovna, Candidate of Physical and Mathematical Sciences, Hirsch Index – 3, Researcher ID 0000-0003-0247, ORCID: 0000-0003-0247-5985, SsrIs author ID: 56747919300.</li> <li>2. Kadirbayeva Zhazira Muratbekovna, Candidate of Physical and Mathematical Sciences, Hirsch Index – 8; Researcher ID : AAN-7014-2020, ORCID: 0000-0001-8861-4100, Scopus Author ID: 57195808858.</li> <li>3. Sandugash Tabyldievna Mynbayeva, PhD, Hirsch Index – 4; Researcher ID : AAN-7014-2020, ORCID: 0000-0001-6266-9357, Scopus Author ID: 57211938645.</li> <li>4. Tokmurzin Zhanibek Syrlybaevich, PhD, Hirsch Index – 2; Researcher ID : AAN-7014-2020, ORCID: 0000-0002-3738-5923 , Scopus Author ID: 57218369903.</li> <li>5. Asan Janela Zheniskyzy, Master, Hirsch Index – 0, ORCID: 0000-0002-3617-2782.</li> </ol>
<p>List of publications with links to them</p>	<ol style="list-style-type: none"> <li>1. Zh. M. Kadirbayeva, S. S. Kabdrakhova, S. T. Mynbayeva A, computational method for solving the boundary value problem for impulsive systems of essentially loaded differential equations. Lobachevskii journal of Mathematics. Volume 42, issue 15, pages 3675–3683 (2021) <a href="https://doi.org/10.1134/S1995080222030131">https://doi.org/10.1134/S1995080222030131</a></li> <li>2. Zh. M. Kadirbayeva, S. S. Kabdrakhova A, numerical solution of problem for essentially loaded differential equations with an integro-multipoint condition. Open Mathematics, Volume 20, issue 1, pages 1173–1183 (2021) <a href="https://doi.org/10.1515/math-2022-0496">https://doi.org/10.1515/math-2022-0496</a></li> </ol>

3. S.S. Kabdrakhova, O.N. Stanzhytskyi, Necessary and sufficient conditions for the well-posed solvability of a boundary value problem for a linear loaded hyperbolic equation. // Journal of Mathematics, Mechanics and Computer Science. Vol.112, №4 (2021) P. 3-12. DOI: <https://doi.org/10.26577/JMMCS.2021.v112.i4.01>
4. Zh. M. Kadirbayeva, On an Algorithm for Solving a Problem with Parameter for the Essentially Loaded Differential Equations Lobachevskii journal of Mathematics, 2022, Vol. 43, No. 11, pp. 3183-3191. <https://doi.org/10.1134/S1995080222140177>
5. S. S. Kabdrakhova, On the existence of the solution of the boundary value problem for linear loaded hyperbolic equations//Abstracts of the reports of the traditional international April Mathematical conference in honor of the Day of Science Workers of the Republic of Kazakhstan, dedicated to the 75th anniversary of Academician of the National Academy of Sciences of the Republic of Kazakhstan T.Sh.Kalmenov, pp.33-34, Almaty 2021.
6. S.S. Kabdrakhova, Algorithm for finding the solution of a semi-periodic boundary value problem for systems of loaded hyperbolic equation. Abstract the international conference: Dynamical systems, Modeling, and Mathematical Sciences. Dubai/ UAE, P. 31, September 23-25, 2022
7. S. S. Kabdrakhova, Zh. Kadirbayeva, A Computer Modeling of Problem for Essentially Loaded Differential Equations with an Integro-Multipoint Condition, P.76-77, International Conference Proceedings, IICAST-22, TCBEM-22, ALHSS-22&TBEEL-22, Vol 1, December 15-16, 2022, Istanbul, Turkey.
8. S. S. Kabdrakhova, Zh. Assan On an approximate method for solving a boundary value problem for a linear loaded hyperbolic equation. //Abstracts of reports

"Traditional international April mathematical conference in honor of the Day of Science workers of the Republic of Kazakhstan". Almaty 2022, pp. 82-83.

9. S. S. Kabdrakhova, Zh. Assan, On a numerical method for solving boundary value problem for a loaded hyperbolic equation. Proceedings IX International Scientific Conference Problems of Differential Equations, Analysis and Algebra, K. Zhubanov Aktobe Regional University, Volume 2, P. 8-10, Aktobe May 24-28, 2022

10. S. S. Kabdrakhova, Zh. Kadirbayeva, Solution of a Family of Boundary Value Problems for Nonlinear Loaded Hyperbolic Equation. XIII International Conference of the Georgian Mathematical Union, Batumi, P.139, September 4 – 9, 2023

11. S. S. Kabdrakhova, A Modification of the Euler polygonal method for solving semi-periodical boundary value problem for loaded nonlinear hyperbolic equation. Mathematics for the Micro/Nano-World: From soliton dynamics, nonlinear optics to quantum science and technology. September 18 – 22, 2023, Samarkand, Uzbekistan. P. 17-18.

12. Kabdrakhova Symbat, Kadirbayeva Zhazira, On one Method for Solving a Boundary Value Problem for a Nonlinear Loaded Ordinary Differential Equation. International Mathematical Conference: Functional Analysis in Interdisciplinary Applications, P.57-58. Antalya, Turkey, October 02-07, 2023.

13. S. S. Kabdrakhova, Zh. Kadirbayeva, Zh. Assan, On one method for solving a semi-periodic boundary value problem for a loaded hyperbolic equation. The first Sharjah International Conference on Mathematical Sciences. University of Sharjah, 6th-8th November 2023, P.111.

14. S.S Kabdrakhova, "Isolated" solution of a boundary value problem for a nonlinear loaded hyperbolic equation.

	<p>Abstracts of the reports of the traditional international April mathematical conference in honor of the Day of the Republic of Kazakhstan. Almaty 2023, 135-136 p.</p> <p>15. S. S. Kabdrakhova Algorithm for finding the solution of a semi-periodic boundary value problem for systems of loaded hyperbolic equation. //Abstract the international conference: Dinamical systems, Modeling, and Mathematical Sciences. Dubai/ UAE, P. 31, September 23-25, 2022</p> <p>16. S.S Kabdrakhova, Conditions for the existence of an “isolated solution” of boundary value problem for a semilinear loaded hyperbolic equation. //Journal of Mathematics, Mechanics and Computer Science. Vol.119, №3 (2023) P. 30-42. DOI: <a href="https://doi.org/10.26577/JMMCS2023v119i3a3">https://doi.org/10.26577/JMMCS2023v119i3a3</a></p> <p>17. S.S Kabdrakhova, Necessary and sufficient conditions for the existence of an "isolated" solution of a semiperiodic boundary value problem for a nonlinear loaded hyperbolic equation //Abstracts of the VII World Congress of Turkic World Mathematicians TWMS Congress 2023, September 20-23, 2023, Turkestan, Kazakhstan. - P. 118.</p> <p>18. M.E. Andirov, S. S. Kabdrakhova, Author's certificate of the Republic of Kazakhstan "Development of an algorithm for finding a solution to a linear two-point boundary value problem for differential equations", Certificate No. 17610 dated May 18, 2021.</p>
Patents	-