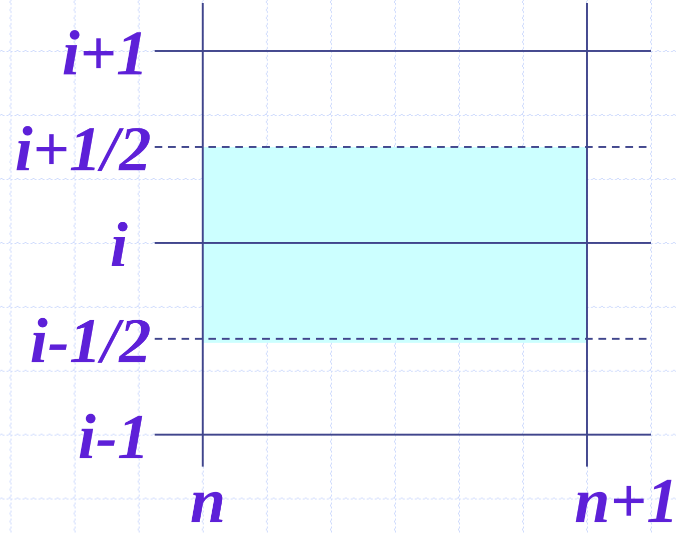
# 3. Метод интегрирования по контрольному объему

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} = a\frac{\partial^2 f}{\partial x^2}$$

$$\int_{i-1/2}^{i+1/2}\int_{n}^{n+1}\frac{\partial f}{\partial t}\,dt\,dx +$$




$$+\int_{n}^{n+1}\int_{i-1/2}^{i+1/2}u\frac{\partial f}{\partial x}\,dx\,dt = \int_{n}^{n+1}\int_{i-1/2}^{i+1/2}a\frac{\partial^2 f}{\partial x^2}\,dx\,dt$$

$$\int_{i-1/2}^{i+1/2}\left(f^{\,n+1} - f^{\,n}\right)dx + u\int_{n}^{n+1}\left(f_{i+1/2} - f_{i-1/2}\right)dt = a\int_{n}^{n+1}\left(\left.\frac{df}{dx}\right|_{i+1/2} - \left.\frac{df}{dx}\right|_{i-1/2}\right)dt$$

# Теорема «о среднем»:

$$\int_{z_k}^{z_{k+1}} f(z)dz \approx f(z^*)\Delta z \qquad \Delta z = z_{k+1} - z_k \qquad z^* \in [z_k, z_{k+1}]$$

$f_{i+1/2} = (f_{i+1} + f_i)/2$

$f_{i-1/2} = (f_{i-1} + f_i)/2$

$x^* = x_i, \quad t^* = t_n$:

$$\left(f_i^{n+1} - f_i^n\right)\Delta x + u\left(f_{i+1/2}^n - f_{i-1/2}^n\right)\Delta t =$$

$$= a\left(\left.\frac{df}{dx}\right|_{i+1/2}^n - \left.\frac{df}{dx}\right|_{i-1/2}^n\right)\Delta t$$

$$\left.\frac{df}{dx}\right|_{i-1/2}^n \approx \frac{f_i^n - f_{i-1}^n}{\Delta x}$$

$$\int_i^{i+1} \left.\frac{df}{dx}\right|^n dx = f_{i+1}^n - f_i^n \approx \left.\frac{df}{dx}\right|_{i+1/2}^n \Delta x \Rightarrow \left.\frac{df}{dx}\right|_{i+1/2}^n \approx \frac{f_{i+1}^n - f_i^n}{\Delta x}$$

точное значение

по т-ме о среднем

$$\left(f_i^{n+1} - f_i^n\right)\Delta x \quad + u\left(\frac{f_{i+1}^n + f_i^n}{2} - \frac{f_{i-1}^n + f_i^n}{2}\right)\Delta t =$$

$$= a\left(\frac{f_{i+1}^n - f_i^n}{\Delta x} - \frac{f_i^n - f_{i-1}^n}{\Delta x}\right)\Delta t$$

$$\frac{\left(f_i^{n+1} - f_i^n\right)\Delta x}{\Delta x\,\Delta t} + u\frac{f_{i+1}^n - f_{i-1}^n}{2}\frac{\Delta t}{\Delta x\,\Delta t} = a\frac{f_{i+1}^n + f_{i-1}^n - 2f_i^n}{\Delta x\ \Delta x\,\Delta t}\Delta t$$

$$\boxed{\frac{f_i^{n+1} - f_i^n}{\Delta t} + u\frac{f_{i+1}^n - f_{i-1}^n}{2\Delta x} = a\frac{f_{i+1}^n + f_{i-1}^n - 2f_i^n}{\Delta x^2}}$$

# Домашнее задание:

**4. Найдите точное и приближенные значения первой и второй производной в точке $x = 1$ следующей функции:**

$$f(x) = \frac{a}{b} e^{-\frac{x}{b}} \qquad \left.\frac{df}{dx}\right|_{x=1} = \left.\frac{a}{b} e^{-\frac{x}{b}}\right|_{x=1} = 2e^{-\frac{1}{10}} = ?$$

$$\left.\frac{d^2 f}{dx^2}\right|_{x=1} = ?$$

$a = 20,\ b = 10,\ \Delta x = 0{,}2$

**Приближенные значения первой производной найдите по трем формулам (левосторонняя, правосторонняя и центральная аппроксимации).**

***Подсказка:***

$f_i = f(x_i) = f(1);$ $\quad f_{i+1} = f(x_i + \Delta x) = f(1+0{,}2) = f(1{,}2)$

$f_{i-1} = f(x_i - \Delta x) = f(1-0{,}2) = f(0{,}8)$