APPROVED
at a meeting of the Academic Council of
NJSC «KazNU named after al-Farabi»
Protocol № 11 from 23.05.2025 y.

The program of the entrance exam for applicants to the PhD
for the group of educational programs
D194 – «Mathematical and computer modeling»

I. General provisions

1. The program was drawn up in accordance with the Order of the Minister of Education and Science of the Republic of Kazakhstan dated October 31, 2018 No. 600 «On Approval of the Model Rules for Admission to Education in Educational Organizations Implementing Educational Programs of Higher and Postgraduate Education» (hereinafter referred to as the Model Rules).

2. The entrance exam for doctoral studies consists of writing an essay, an exam in the profile of a group of educational programs and an interview.

Блок	Баллы
1. Interview	30
2. Essay	20
3. Exam according to the profile of the group of the educational program	50
Total admission score	100/75

3.The duration of the entrance exam is 3 hours 10 minutes, during which the applicant writes an essay and answers the electronic examination ticket. The interview is conducted at the university premises before the entrance exam.

II. Procedure for the entrance examination

1. Applicants for doctoral studies in the group of educational programs D194 – «Mathematical and computer modeling» write a problematic / thematic essay. The volume of the essay is at least 250words.

The purpose of the essay is to determine the level of analytical and creative abilities, expressed in the ability to build one's own argumentation based on theoretical knowledge, social and personal experience.

Types of essays:

- motivational essay revealing the motivation for research activities;
- scientific-analytical essay justifying the relevance and methodology of the planned research;
- problem/thematic essay reflecting various aspects of scientific knowledge in the subject area.

2. The electronic examination card consists of 3 questions

Topics for exam preparation according to the profile of the group of the educational program:

Course: "Fundamentals of Mathematical Modeling"

Fundamental principles of constructing mathematical models. Mathematical modeling and its areas of application. Characteristics of a mathematical model. Main approaches to mathematical modeling. Creating a mathematical model based on fundamental laws of nature. Constructing mathematical models using variational principles. Hierarchical approach to model construction, illustrated by the example of a three-stage rocket. Bottom-up model hierarchy using the example of a mechanical system and features of its solution. Application of the analogy method in constructing models from various fields of science (e.g., the radioactive decay model and the Malthusian model; pendulum oscillation and electrical circuit models, etc.). Modeling economic problems. Advertising campaign model. Macroeconomic model of a country's economic growth. Modeling problems in law and state governance. Linear and nonlinear mathematical models. Three regimes of a nonlinear population model. Phase portraits. Modeling of continuous media. Closure of the mass conservation law for gravitational groundwater flow. Darcy's law. Some properties of the Boussinesq equation. Hierarchy of groundwater flow models. Main requirements for mathematical models. Dimensional analysis of models. Model non-dimensionalization. Classification of mathematical models depending on the type of model operator. Main types of equations in mathematical models. Application software packages and their classification.

Course: "Mathematical and Computer Modeling of Physical Processes"

The Thomas algorithm (tridiagonal matrix algorithm). Simple iteration method for the heat conduction equation. Stability conditions. Five-point Thomas algorithm. Compact scheme. First upwind scheme. Second upwind scheme. Matrix Thomas algorithm. Jacobi, Gauss-Seidel, and Successive Over-Relaxation (SOR) methods. Fractional step method (FSM) for the two-dimensional heat conduction equation. Alternating direction implicit (ADI) method for the two-dimensional heat conduction equation. Fractional step method (FSM) for the three-dimensional heat conduction equation. Alternating direction implicit (ADI) method for the three-dimensional heat conduction equation. Fourier method for the three-dimensional Poisson equation. Mathematical modeling of atmospheric processes. Mathematical modeling of ocean pollution. Mathematical modeling of natural convection. Mathematical modeling of internal flows. Mathematical modeling of wind flow around man-made obstacles. Mathematical modeling of flow separation behind a backward-facing step in a channel. Reynolds equation.

Course: "Mathematical and Computer Modeling of Medical and Biological Processes"

Dynamical systems. Basic concepts and properties of dynamical systems. Phase portraits and trajectories. One-dimensional systems. Analysis of equilibrium points in two-dimensional linear dynamical systems. The concept of the characteristic equation. Stability of equilibrium points. Lyapunov’s direct method. Stability and instability in the first approximation. Lyapunov’s theorem (Sylvester’s criterion). Routh–Hurwitz criterion. Non-zero equilibrium points of dynamical systems. Population dynamics. Exponential growth model, logistic equation. Models of interaction between two species. Generalized models with specified functions. Models involving biochemical reactions. Laws of mass conservation. Enzyme kinetics. Morphogenesis models in reaction–diffusion systems. The Brusselator model. Spatial models of morphogenesis in reaction–diffusion systems. Numerical modeling of “leopard skin” pattern formation in a reaction–diffusion system.

III List of references

Main:

1. Самарский, А.А. Математическое моделирование. Идеи, методы, при- меры / А.А. Самарский, А.П. Михайлов. – М.: Физматлит, 2002.– 315 с.
2. Мышкис, А.Д. Элементы теории математических моделей. – М.: Ком- Книга, 2007. – 192 с.
3. Трусов, П.В. [и др.] Введение в математическое моделирование. – М.: Logos, 2005. – 440 с.
4. Трусов, П.В. [и др.] Введение в математическое моделирование. – М.: Logos, 2005. – 440 с.
5. Седов, А.И. Методы подобия и размерности в механике. – М.: Наука, 1977. – 440 с.
6. Ли, Софус. Теория групп преобразований: Ч. I. – М. – Ижевск: Ижев- ский ин-т комп. иссл., 2011. – 712 с.
7. Овсянников, Л.В. Групповые свойства дифференциальных уравнений.Новосибирск: Изд-во СОАН СССР, 1962. – 242 с.
8. Владимиров, В.С. Уравнения математической физики. – М.: Наука, 1981. – 512 с.
9. Тихонов, А.Н. Методы решения некорректных задач / А.Н. Тихонов, В.Я. Арсенин. – М.: Наука, 1979. – 283 с.
10. Новожилов, В.В. Основы нелинейной теории упругости. – М. – Л.: ОГИЗ, 1948. – 211 с.
11. Хаджиева, Л.А. Нелинейные модели динамических систем: учеб. пособие. – Алматы: Қазақ университеті, 2009. – 133 с.
12. Бордовский Г. А. Кондратьев А.С. и др. Физические основы математического моделирования / М. ACADEMIA, 2005. -321 с.
13. Муканова Б.Г., Хаджиева Л.А. Математикалық модельдеуге кіріспе. -Алматы: Қазақ университеті, 2019. – 228 б.
14. Бейли Н. Математика в биологии и медицине. Москва, Изд-во «Мир», 1970
15. Марри Дж. Нелинейные дифференциальные уравнения в биологии. Лекции о моделях. Москва, «Мир», 1983
16. Дж. Смит Математические идеи в биологии. Москва, Изд-во «Мир», 1970
17. Базыкин А.Д. Математическая биофизика взаимодействующих популяций. Москва, Изд-во «Наука», 1985 г.
18. Белоцерковский О.М. Численное моделирование в механике сплошных сред. Москва, Изд-во «Наука», 1984 г.
19. Славин М.Б. Методы системного анализа в медицинских исследованиях.- М.: Медицина, 1989.- 302 с.
20. Ризниченко Г.Ю., Рубин А.Б. Математические модели биологических продукционных процесов. М., Изд. МГУ, 1988
21. http://www.biometrica.tomsk.ru/annot6.htm
22. http://www.biometrica.tomsk.ru/beili.htm
23. Жумагулов Б.Т., Абдибеков У.С., Исахов А.А. Основы математического и компьютерного моделирования естественно-физических процессов. Алматы, «Қаза университеті», 2014. –208 с.
24. Исахов А.А. Практикум по математическому и компьютерному моделированию физических процессов. – Алматы: Қазақ университеті, 2015;
25. Марчук Г.И. Математическое моделирование в проблеме окружающей среды. –М.: Наука, 1982. –320 с.
26. Яненко Н.Н. Метод дробных шагов решения многомерных задач математической физики. –Новосибирск: Наука, 1967. – 196 с.
27. Флетчер К. Вычислительные методы в динамике жидкости. - М.: Мир, 991. – Т. 2 – 552 с.
28. Роуч П. Вычислительная гидродинамика. М.: Мир, 1980. - 616 с.
29. Пейре Р., Тейлор Т.Д. Вычислительные методы в задачах механики жидкости. –Л.: Гидрометеоиздат, 1986. - 352 с.

30. Самарский А.А. Теория разностных схем. –М.: Наука, 1989. – 616 с.
31. Иевлев В.М. Численное моделирование турбулентных течений. М.:Наука, 1990. - 273 с.
32. Курбацкий А.Ф. Моделирование нелокального турбулентного переноса импульса и тепла. - Новосибирск: Наука, 1988. - 240 с.

Additional:

1. Пинегина, М.В. Математические методы и модели в экономике. – М.: Экзамен, 2004. – 128 с.
2. Тихонов, А.Н. Уравнения математической физики / А.Н. Тихонов, А.А. Самарский - М.: Наука, 1999. – 799 с.
3. Годунов С.К. Уравнения математической физики. – М.: Наука, 1979.
4. – 392 с.
5. Ладыженская, О.А. Краевые задачи математической физики. – М.: Наука, 1973. – 409 с.
6. Беллман Ричард. Математические методы в медицине / Пер. с англ. А.Л.Лисаченкова, И.Л.Шалькова; под ред. Белых.- М.: Мир, 1987.- 200 с.
7. И.Б. Петров Математическое моделирование в медицине и биологии на основе моделей механики сплошных сред ТРУДЫ МФТИ, 2009, Том 1, № 1 с5-16
8. http://www.apmath.spbu.ru/ru/education/courses/special/semko1.html